

**Unit 5: Average Value**

**Definition:** Let  $f$  be a function which is continuous on the closed interval  $[a, b]$ .  
The **average value** of  $f$  from  $x = a$  to  $x = b$  is the integral

Average Value:

**Examples:**Find the average value of  $f$  on the given interval.

1.  $f(x) = x^2 + 2x$  on  $[0,3]$

2.  $f(x) = \sin x$  on  $[0,\pi]$

3.  $f(x) = e^x$  on  $[0,2]$

4.  $f(x) = \frac{1}{x}$  on  $[1,4]$

## Unit 5: Mean Value Theorem for Integrals

**REVIEW: MVT for Differentiation**

If  $f$  is continuous on a closed interval  $[a,b]$  and differentiable on its interior  $(a,b)$ ,

then there is at least one number  $c$  in  $(a,b)$  where: 
$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Example:  $f(x) = x^2 - 4x + 5$   $[0,6]$

Mean Value Theorem for Integrals:

If  $f$  is continuous on  $[a,b]$ , then there exists a number  $c$  in  $[a,b]$  such that

$$\int_a^b f(x)dx = f(c)(b - a) \quad \text{OR} \quad f(c) = \text{average value of } f(x)$$

Examples:

1. Apply the MVT for Integrals for  $f(x) = 1 + x^2$  on  $[-1,2]$
  
2. Apply MVT for integrals for  $f(x) = \sin x$  on  $[0,\pi]$ .
  
3. Find the value of  $b$  such that the average value of  $f(x) = 2 + 6x - 3x^2$  on the interval  $[0,b]$  equals 3.